

## MORTALITY MODELS

These nine files are used to construct mortality models (\*) which consider the sex: female, male or unisex, and the type of mortality: high, medium or low.

With the programs presented, we intend to offer an instrument which allows us, not only to obtain the requested mortality models. Each model is completed with several indicators that permit the estimation of respective explanatory capacities.

The window opened on this subject allows us to see the structure of the operations to follow for obtaining mortality models.

The main characteristics of this type of life table model are:

**1) the mortality** models are based on an analytical function (the resistance function) which also provides a particular way of describing and interpreting the process of mortality by age class. In a life-table  $l_x$  survivors at age  $x$  (variable between zero at birth, and  $\omega$  at the extreme age of life), are inserted in a process which foresees progressive eliminations until mortality reaches the point where  $l_\omega = 0$ .

Starting from the initial age zero onwards, the mortality process is contrasted by the reaction of the survivors at the different ages which contrapose a resistance tending to reduce the final moment of the process and the elimination of those belonging to the residual group.

Supposing that the survival function is continuous and derivable, let us propose the function of resistance as follows:

$$r(x) = \frac{x l(x)}{(\omega - x)[l(0) - l(x)]} \quad [1]$$

The value of  $r(x)$  at each age  $x$  represents the number of deaths which on average will occur between  $x$  and  $\omega$ , for each death observed between birth and  $x$ .

In these models, the maximum age expectancy is fixed at 110 years, which can be considered as its possible biological limit in our time.

The survival function derived from [1], with  $l(0)=1$ , is:

$$p(x) = \frac{1}{x[(\omega - x)r(x)]^{-1} + 1} \quad [2]$$

whose force of mortality is:

$$\mu(x) = [1 - p(x)] \left[ \frac{1}{x} + \frac{1}{\omega - x} - \frac{d r(x)}{dx} \cdot \frac{1}{r(x)} \right] \quad [3]$$

Having represented the function of resistance as follows:

$$r(x) = x^{P(1)} (\omega - x)^{P(2)} e^{P(3)x^2 + P(4)x + P(5)} \quad [4]$$

with the parameters  $P(1)$ ,  $P(2)$ ,  $P(3)$ ,  $P(4)$  and  $P(5)$ .

Therefore, fixed  $l(0)=100000$  births, the survival function results:

$$l(x) = \frac{100000}{x^{1-P(1)} (\omega - x)^{-[1+P(2)]} e^{-P(3)x^2 - P(4)x - P(5)} + 1} \quad [5]$$

The bell trend in the resistance distribution has demonstrated the existence of 5 points which represent the examined mortality situation, that is:  $r(1)$  and  $r(10)$ , the maximum value of resistance  $r(XM)$  with the corresponding age  $XM$ , and  $r(75)$ .

Using the  $r(x)$  values from the 290 survival tables utilized as a technical basis, some interesting relationships emerge between the above mentioned  $r(x)$  points, demonstrating a high degree of correlation.

These correlation functions serve in setting up the following system, the solution of which allows us to obtain the values of the parameters  $P(1)$ ,  $P(2)$ ,  $P(3)$ ,  $P(4)$  and  $P(5)$ , which define both the  $r(x)$  function and the survival function  $l(x)$ :

**2) the models** are defined for each sex and both sexes combined, using an identical methodology;

**3) there** are no group models describing particular geographical areas or life conditions of the population considered (for example, developed or underdeveloped countries). The model tables constitute a single scheme, since the differentiation of the general mortality of man is considered to be more due to environmental conditions than to race.

The regularity observed in the correlation between some biometric indicators of the observed life tables has been expressed by the model patterns at "medium mortality", "low mortality" and "high mortality".

**4) the following** values are obtained from each model: the survivors; the fraction of the last interval of age lived; the age specific death rates; the probability of dying, the stationary and stable population; the resistance function; the expectation of life at various ages; the age distribution of the population; the deaths distribution by age; death rates and birth rates; the age-child ratio; the dependency ratio; an estimation of the total fertility rate.

The values of the various biometrical functions can be determined for each exact age for different age groups from birth to the extreme age;

**5) a great** variety of models can be produced (within the limits imposed by the formal conditions guiding the procedure for model construction) from one entry parameter, which in this program is the infant mortality rate.

Although there is an ample selection for the values of XM, caution is necessary when using values higher than XM. Especially for ages over 85-90 years, a stationariness of the values of the resistance function  $R(x)$  may appear, leading to a slowing down of the mortality.

In this situation, it is appropriate to choose values for XM which are centrally positioned within the range of values presented for any model.

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(\*) A synthesis of the theory and characteristics of these models is explained in the following works:

Petrioli.L. (1981), "A new set of models of mortality", Seminar IUSSP, Dakar, Senegal, July 7-10.

Petrioli.L. (1982), "Nouvelle tables-types de mortalité. Application et population stable", Séminaire au Département de Démographie de l'Université da Kinshasa, Rep.Dem.du Congo (ex-Zaire),15-12 décembre.

Petrioli.L. (1998), "Demografia. Fatti e metodi di studio della popolazione",F.Angeli Editor Milan, Italy.

## DESCRIPTION OF THE FUNCTIONS CALCULATED

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BR = BIRTH RATE  
CON = CONVEXITY OF THE LIFE-TABLE BY ANNUAL AGE-CLASSES  
C(X) = DISTRIBUTION OF THE STABLE POPULATION AT AGE X  
DR = DEATH RATE  
D(X) = DISTRIBUTION OF DEATHS FROM 100000 BIRTHS  
EO = EXPECTATION OF LIFE AT BIRTH  
E(X) = EXPECTATION OF LIFE AT AGE X  
H(X) = FRACTION OF THE LAST INTERVAL OF AGE LIVED  
ID = DEPENDENCY RATIO  $(P(65-)+P(0-14))/P(0-14)$ PER 100  
L(X) = STATIONARY POPULATION OF THE TABLE AT AGE X  
M(X) = AGE SPECIFIC DEATH RATE BETWEEN EXACT AGE X AND X+H  
Q(0) = PROBABILITY OF DYING IN THE FIRST YEAR OF LIFE  
Q(X) = PROBABILITY OF DYING BETWEEN EXACT AGE X AND X+H  
R(X) = FUNCTION OF RESISTANCE AT AGE X  
ST(X) = STABLE POPULATION AT AGE X  
TA = RATE OF POPULATION GROWTH  
TFR = TOTAL FERTILITY RATE  
XM = AGE OF MAXIMUM RESISTANCE  $R(X)$