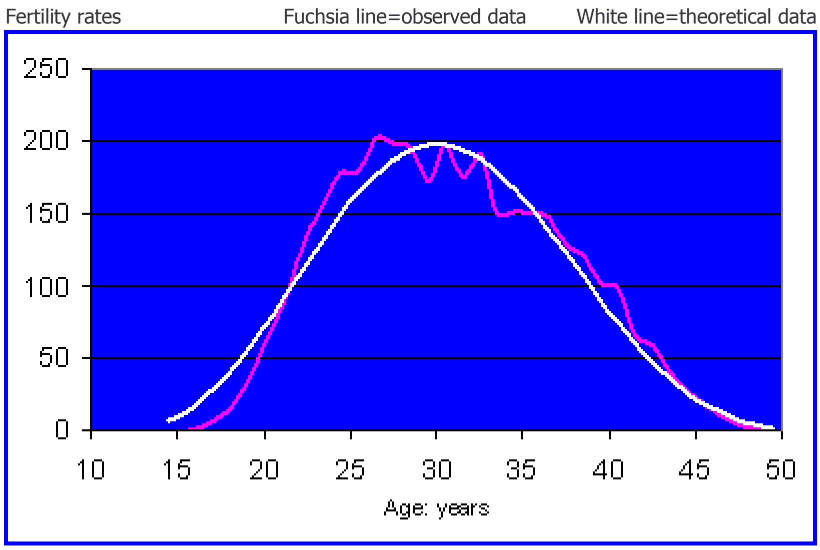
FERTILITY FUNCTIONS

(Source:Petrioli Luciano,"*PRODEMOG 3.0-Demographic software for Windows*", EMMECI-SIENA-ITALY,(2000).

B E T A



Age-specific fertility rates: Italy, year 1930

The density function for Beta is:

$$f(X) = C\left(\,.\Gamma(A^{*}+B^{*})/[\Gamma(A^{*})\cdot\Gamma(B^{*})]\cdot\ \right)X^{A^{*}-1}(1-X)^{B^{*}-1}\cdot \qquad [1]$$

with the parameters A* ,B* >0 e 0≤ X ≤ 1.
In order to use the independent age variable, which in the fertility distributions can be set between 10 and 50 years (or other limits), a new variable ***t*** is set considering that it is the real and observed age x, from the extreme ages in the field of variation chosen, that is ***t*** = (X-U)/(V-U). It follows that t assumes a zero value for X=U, and a value of one for X =V , having determined:
U = lower limit of fertility period:
V = upper limit of fertility period;
X = one of the values included between U and V.
Also keeping in mind that the value of the total fertility rate TFT (indicated from now on as C) which we develop in the interval U,V and not between zero and one, the function elaborated by program BETA is written:

$$f(x) = \frac{C}{(V-U)} \cdot \frac{\Gamma(A+B)}{\Gamma(A)\cdot\Gamma(B)} \cdot \left(\frac{X-U}{V-U}\right)^{A-1} \cdot \left(1-\frac{X-U}{V-U}\right)^{B-1} \quad [2]$$

It is also necessary to consider that as a result of the transformation of variables performed, we have the value of the mean (MEDT) and of the empirical variance (DST) with respect to x (MED, DS) for which we obtain the following relations:

$$A = \frac{MEDT^2(1-MEDT)}{DST} - MEDT;$$

$$B = \frac{(1-MEDT)^2 \cdot MEDT}{DST} - (1-MEDT) \quad [3]$$

The approximated values of $\Gamma(Z)$ are calculated by using the formula (*) :

$$\Gamma(Z) = \sqrt{2\pi / Z}.\, Z^Z.\, e^{-Z+1/(12Z)}$$

which, for example, in determining the value of Γ(A+B)= Γ(12.74835) in application to the table for Italy-1960, gives the results of 254319840. Considering, instead, the relation:
Γ(n+z) = (n-1+z)(n-2+z)(n-3+z)......(1+z)Γ(1+z)
that is, in this case, making:
Γ(12.74835)=(11.74835)(10.74835)(9.74835).....(1.74835)Γ(1.74835)= 254296856.41

(*) For bibliographic references regarding the described procedures and numeric tables relative to the Gamma function, see:

a) Keyfitz. N. and Flieger. W. (1971), " *Population. Facts and methods of Demography*", Freeman &Company, S. Francisco, USA.

b) M. Abramowitz and I. A . Stegun (1972), "*Handbook of Mathematical Functions*", New York, Dover.